On the effect of cosmological inflow on turbulence and instability in galactic discs

Shy Genel[1,2]⋆, Avishai Dekel[3], Marcello Cacciato[3]

[1] *Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138, USA*
[2] *School of Physics and Astronomy, Tel Aviv University, Tel Aviv 69978, Israel*
[3] *Racah Institute of Physics, The Hebrew University, Jerusalem 91904, Israel*

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ABSTRACT
We analyse the evolution of turbulence and gravitational instability of a galactic disc in a quasi-steady state governed by cosmological inflow. We focus on the possibility that the coupling between the in-streaming gas and the disc is maximal, e.g., via dense clumps, and ask whether the streams could be the driver of turbulence in an unstable disc with a Toomre parameter $Q \sim 1$. Our fiducial model assumes an efficiency of ~ 0.5 per dynamical time for the decay of turbulence energy, and ~ 0.02 for each of the processes that deplete the disc gas, i.e., star formation, outflow, and inflow within the disc into a central bulge. In this case, the in-streaming drives a ratio of turbulent to rotation velocity $\sigma/V \sim 0.2 - 0.3$, which at $z \sim 2$ induces an instability with $Q \sim 1$, both as observed. However, in conflict with observations, this model predicts that σ/V remains constant with time, independent of the cosmological accretion rate, because mass and turbulence have the same external source. Such strongly coupled cosmological inflow tends to stabilize the disc at low z, with $Q \sim$ a few, which may be consistent with observations. The instability could instead be maintained for longer, with a properly declining σ/V, if it is self-regulated to oscillations about $Q \approx 1$ by a duty cycle for disc depletion. However, the 'off' phases of this duty cycle become long at low z, which may be hard to reconcile with observations. Alternatively, the coupling between the in-streaming gas and the disc may weaken in time, reflecting an evolving nature of the accretion. If, instead, that coupling is weak at all times, the likely energy source for self-regulated stirring up of the turbulence is the inflow within the disc down the potential gradient (studied in a companion paper).

Key words: galaxies: evolution – galaxies: formation – galaxies: high-redshift – galaxies: kinematics and dynamics – galaxies: star formation – methods: analytical

1 INTRODUCTION

The basic kinematical properties of galaxies are the average rotation velocity $V_{\rm rot}$, and the velocity dispersions σ of their components, namely the random motions of stars and the gas turbulence in the interstellar medium (ISM). Disc galaxies are supported against gravity by rotation. At low redshift, massive discs have gas velocity dispersions of $\sim 10\,{\rm km s^{-1}}$ (Dib et al. 2006), with $\sigma/V_{\rm rot} \approx 0.05 - 0.1$. Their stellar velocity dispersion is typically at the level of tens of km s^{-1} and it varies with stellar surface density (Bottema 1993; van der Kruit 2010; Westfall et al. 2011). Observations show larger gas velocity dispersions at higher redshifts (Epinat et al. 2010; Davies et al. 2011), such that typical massive disc galaxies at $z \approx 2$ have $\sigma \approx 30 - 80\,{\rm km s^{-1}}$ and

$\sigma/V_{\rm rot} \approx 0.15 - 0.3$ (Erb et al. 2004; Förster Schreiber et al. 2006; Cresci et al. 2009).

Galactic discs are often assumed to maintain marginal gravitational instability, with a Toomre Q parameter $Q \sim 1$ (Toomre 1964). In this case,

$$\frac{\sigma}{V_{\rm rot}} \approx (2\nu)^{-1/2}\delta \,, \tag{1}$$

where δ is the mass fraction in cold disc within the disc radius,

$$\delta \equiv \frac{M_{\rm disc}}{M_{\rm tot}} \,, \tag{2}$$

and ν is a factor of order unity that depends on the shape of the rotation curve,

$$\nu \equiv 1 + \frac{\log V_{\rm rot}}{\log r} \simeq 1 \tag{3}$$

⋆ E-mail: sgenel@cfa.harvard.edu

(Dekel et al. 2009b). Eq. (1) explains why at high redshift the high fraction of cold disc, in terms of gas and young stars, requires a high value of $\sigma/V_{\rm rot}$ in order to maintain $Q \sim 1$. The instability of discs with high δ and $\sigma/V_{\rm rot}$ is more 'violent', in the sense that the structures are larger and the dynamical processes are faster (Genzel et al. 2008; Dekel et al. 2009b; Genzel et al. 2011; Ceverino et al. 2012; Bournaud et al. 2011a,b). The perturbations associated with the instability, in the form of extended transient features and bound clumps, reflect the larger characteristic Toomre mass,

$$\frac{M_{\rm clump}}{M_{\rm disc}} \simeq \frac{1.2}{\nu} \left(\frac{\sigma}{V_{\rm rot}} \right)^2 . \qquad (4)$$

Torques between the perturbations drive angular momentum out and generates mass inflow, partly as clump migration and partly as inflow of inter-clump mass. The timescale for inflow can be estimated in several different ways (Gammie 2001; Dekel et al. 2009b) to be

$$\frac{t_{\rm inf}}{t_{\rm dyn}} \simeq 5 \left(\frac{V_{\rm rot}}{\sigma} \right)^2 , \qquad (5)$$

where

$$t_{\rm dyn} \equiv \Omega^{-1} = \frac{R}{V_{\rm rot}} , \qquad (6)$$

Ω is the angular velocity and R is the effective radius of the disc. Thus, a higher $\sigma/V_{\rm rot}$ is associated with a faster inflow in the disc.

Since ISM turbulence decays on one or a few dynamical timescales (Mac Low et al. 1998; Stone et al. 1998; Gammie 2001; Elmegreen & Scalo 2004), there must exist a continuous energy source that maintains the turbulence over cosmic time. However, the nature of this energy source is highly debatable. The mechanisms that drive the ISM turbulence could be divided into three kinds. First, *stellar feedback*, such as supernova feedback and radiative feedback from stars, which deposit energy and momentum into the ISM. Second, the energy source could be the gravitational energy released by the instability-driven inflow down the potential gradient within the disc, which is a natural mechanism for self-regulating the instability at $Q \sim 1$ (Wada et al. 2002; Agertz et al. 2009; Bournaud et al. 2010; Krumholz & Burkert 2010; Cacciato et al. 2011). Third, the driver of turbulence could be the kinetic energy transferred from the cosmological inflow of clumpy gas. At high redshift, the gas streams in as supersonic streams that follow the filaments of the cosmic web. The streams, consisting of merging galaxies and a smoother component, penetrate through the halo to the vicinity of the central disc where they deposit a fraction of their energy and momentum (Birnboim & Dekel 2003; Kereš et al. 2005; Dekel & Birnboim 2006; Kereš et al. 2009; Dekel et al. 2009a). At low redshift, the flow consists of cold gas clouds that 'rain' from the hot halo onto the disc (Maller & Bullock 2004; Dekel & Birnboim 2008; Kereš & Hernquist 2009). Such flows may be efficiently converted into turbulence (Klessen & Hennebelle 2010), provided that the in-streaming gas has comparable density to the disc ISM (e.g. Dekel et al. 2009b).

The role played by stellar feedback in driving the ISM turbulence is controversial. On one hand, theoretical estimates and numerical simulations argue that stellar feed-back could be the main driver of turbulence in local massive galaxies (Mac Low & Klessen 2004; Kim et al. 2001; Dib et al. 2006; de Avillez & Breitschwerdt 2007) as well as in high-redshift galaxies (Hopkins et al. 2011). On the other hand, other estimates and simulations argue that stellar feedback is unlikely to drive a velocity dispersion larger than $\approx 10 \,{\rm km s}^{-1}$ (Joung et al. 2009; Dekel et al. 2009b; Bournaud et al. 2010; Ostriker & Shetty 2011).

Unlike stellar feedback, where the energy emerges from nuclear processes inside stars, both the second and third mechanisms refer to 'gravitational heating', where gravitational potential energy is released as a result of infall into a potential well (Dekel & Birnboim 2008; Khochfar & Ostriker 2008). The second mechanism, based on the inflow within the disc, is determined by the self-regulated disc instability, and is the topic of a companion paper by Cacciato et al. (2011). The third, based on mass streaming in from outside the disc, represents an external source of energy that is determined by the cosmic growth of structure and is independent of the disc instability. In this work we focus on this external mechanism.

So far, different authors seem to have reached different conclusions concerning the possible role played by these 'gravitational heating' mechanisms at $z \sim 2$ (Genzel et al. 2008; Khochfar & Silk 2009; Lehnert et al. 2009; Elmegreen & Burkert 2010; Klessen & Hennebelle 2010; Krumholz & Burkert 2010). This uncertainty naturally stems from the fact that the power provided by cosmological in-streaming is in the same ball park as the turbulence dissipation rate. This motivates the more detailed analysis presented in this paper.

The evolution of the gas mass in a disc galaxy can be described as a quasi-steady-state solution of a simple differential equation of mass conservation (Finlator & Davé 2008; Dekel et al. 2009b; Bouché et al. 2010; Dutton et al. 2010; Davé et al. 2011a,b, see below). In turn, the generation of turbulence and its dissipation in a steady state is governed by an analogous equation of energy conservation (e.g. Khochfar & Silk 2009; Elmegreen & Burkert 2010; Klessen & Hennebelle 2010). Together, these equations help constrain the disc instability, as the Q parameter depends on disc mass and turbulence.

There is numerical and observational evidence for marginal instability with $Q \sim 1$ in disc galaxies. Simulations reveal $Q \approx 1$ in high-redshift, gas-rich discs (Immeli et al. 2004; Ceverino et al. 2010; Genel et al. 2012; Hopkins et al. 2011), and $Q \approx 2-3$ in low-redshift stellar-dominated discs (Hohl 1971; Athanassoula & Sellwood 1986; Bottema 2003; Martig et al. 2009). Similar estimates are obtained from observed discs, both in local galaxies (Leroy et al. 2008; Westfall et al. 2011; van der Kruit & Freeman 2011; Yim et al. 2011; Watson et al. 2012) and in $z \sim 2$ discs (Genzel et al. 2011). However, the parameter Q applies to linear perturbation theory and caution is required in comparing it with estimates from the high non-linearly evolved (observed and simulated) galaxy discs.

The instability of a disc and the associated level of turbulence may be coupled via a self-regulation mechanism (e.g., Dekel et al. 2009b). If the turbulence is driven by internal processes that themselves depend on the disc instability, such as star formation or internal torques that cause mass inflow, the system may relax into a steady-state by a self-

regulation loop. In this case the disc maintains marginal instability, $Q \sim 1$, as the turbulence adjusts itself to the proper value dictated by the gas surface density and the angular velocity. In a companion paper (Cacciato et al. 2011, see also Krumholz & Burkert 2010), we impose $Q = 1$, and analyse the steady state solution of the mass and energy equations under the assumption that the energy source for driving the turbulence is the inflow down the potential gradient within the disc. The rate of this inflow adjusts itself to compensate for the turbulence dissipative losses such that $Q = 1$ is maintained. In Cacciato et al. (2011) we address the instability of a two-component disc, with gas and stars of different velocity dispersions that gradually exchange mass. We find there that discs tend to 'stabilize' at low redshift as the disc becomes dominated by the 'hot' stellar component. Forbes et al. (2011) study a similar scenario including radial variations within the disc.

In this paper, we study the steady-state solution of similar mass and energy conservation equations, but focus on an external energy source, carried by the cosmological in-streaming of gas. We address two main cases. First, a case where the system is governed by the external source alone and the instability is not self-regulated, and where the efficiencies of the various physical processes are constant in time. Second, an alternative case where the instability is self-regulated by a duty-cycle for instability and star formation.

This paper is organized as follows. In Section 2 we present the equations of mass and energy conservation and their steady-state equations, and introduce our parameterization of the relevant physical scenarios. In Section 3 we investigate the non-self-regulated case I, with fixed efficiencies of the physical processes, and predict the evolution of Q and $\sigma/V_{\rm rot}$. In Section 4 we study the self-regulated case II, where we impose $Q = 1$ and introduce a duty cycle for instability and star formation. In Section 5 we put our results in the context of other results from the literature. In Section 6 we conclude and discuss our results.

2 EQUATIONS FOR MASS AND ENERGY CONSERVATION

In this section we present the basic equations for conservation of mass and turbulent energy in a gaseous galactic disc. Source and drain terms for gas mass and gas turbulent energy are identified. As a result, the gas mass and gas velocity dispersion that characterizes the turbulence are computed in a steady-state solution as a function of the incoming supply rate of cosmological gas and the parameters that characterize the various relevant physical processes.

2.1 The backbone steady state model

The basic equation for the gas mass budget of a galactic disc is

$$\dot{M}_{\rm g} = \dot{M}_{\rm cosmo} - \dot{M}_{\rm sink}, \tag{7}$$

where $M_{\rm g}$ is the disc gas mass, $\dot{M}_{\rm cosmo}$ is the external source term that represents the cosmological gas inflow rate, and $\dot{M}_{\rm sink}$ is the sum of different kinds of 'sinks' that empty the disc of its gas, including star formation, galactic outflows,

and inflows inside the disc into the bulge. Eq. (7) has a very simple and instructive solution if the sink terms can be written as $\dot{M}_{\rm sink} = M_{\rm g}\tau^{-1}$, i.e. if they are proportional to the gas mass itself with a 'sink timescale' proportionality factor[1]. If $\dot{M}_{\rm cosmo}$ and τ vary on a timescale longer than τ, the solution is

$$M_{\rm g} = \dot{M}_{\rm cosmo}\tau(1 - e^{-t/\tau}) \tag{8}$$

$$\dot{M}_{\rm g} = \dot{M}_{\rm cosmo}e^{-t/\tau}. \tag{9}$$

For $t \gg \tau$, it reduces to a steady state solution with

$$\dot{M}_{\rm sink} \approx \dot{M}_{\rm cosmo} \tag{10}$$

$$\dot{M}_{\rm g} \approx 0 \tag{11}$$

$$M_{\rm g} \approx \dot{M}_{\rm cosmo}\tau, \tag{12}$$

in which the sink term $\dot{M}_{\rm sink}$ adjusts itself to match the external source term $\dot{M}_{\rm cosmo}$ (see Bouché et al. 2010). The range of validity of this solution is discussed in detail in Appendix A.

Assuming that the disc has reached the steady state, we use the results from mass and energy conservation to derive the turbulent velocity as follows. We start by considering, for simplicity, only the star formation (SF) part in the gas mass sink term, in the form

$$\dot{M}_{\rm SF} = \frac{M_{\rm g}}{t_{\rm SF}}, \tag{13}$$

and obtain from mass conservation

$$\dot{M}_{\rm SF} = \dot{M}_{\rm cosmo}, \tag{14}$$

and

$$M_{\rm g} = \dot{M}_{\rm cosmo}t_{\rm SF}. \tag{15}$$

In analogy, for the turbulent energy $E_{\rm turb}$ we consider a sink term in the form of a dissipation rate,

$$\dot{E}_{\rm dis} = \frac{E_{\rm turb}}{t_{\rm dis}}. \tag{16}$$

This leads in steady state, in analogy to eq. (14), to

$$\dot{E}_{\rm dis} = \dot{E}_{\rm cosmo}, \tag{17}$$

where $\dot{E}_{\rm cosmo}$ is the rate of in-streaming energy. The analogue to eq. (15) is then

$$E_{\rm turb} = \dot{E}_{\rm cosmo}t_{\rm dis}. \tag{18}$$

We approximate $E_{\rm turb} \approx M_{\rm g}\sigma^2$ and $\dot{E}_{\rm cosmo} \approx \dot{M}_{\rm cosmo}V_{\rm in}^2$, where the in-streaming velocity $V_{\rm in}$, as well as the rotational velocity $V_{\rm rot}$, are assumed to be comparable to the virial velocity of the halo (Dekel et al. 2009a), and the conversion of in-streaming kinetic energy to disc turbulence is assumed to be efficient. Eqs. (15) and (18) then yield

$$\frac{\sigma}{V_{\rm rot}} = \sqrt{\frac{t_{\rm dis}}{t_{\rm SF}}}. \tag{19}$$

A very interesting feature of eq. (19) is that $\sigma/V_{\rm rot}$ turns out to be *independent of the cosmological accretion rate itself*. This unique feature stems from the facts that (a) in steady state both the sinks of SFR and dissipation rate adjust themselves to the corresponding supply rates, (b) in our

[1] As we discuss later, τ is related to the dynamical time of the disc.

current model, the cosmological supply is a common source for both the mass and turbulent energy of the disc, such that the cosmological input always provides the same *specific* turbulent energy, and (c) this supply is determined externally, independent of the conditions in the disc. The implication of this special property of our current model is that $\sigma/V_{\rm rot}$ is expected to be invariant under variations in the mass input rate, which is probably the main source of variation in the galaxy properties related to disc instability, both between different galaxies and as a function of time in the history of each individual galaxy.

Both timescales in eq. (19) are expected to be related to the dynamical time of the disc,

$$t_{\rm dis} \equiv \gamma_{\rm dis} t_{\rm dyn} \,, \tag{20}$$

with $\gamma_{\rm dis}$ a constant parameter with a likely value in the range $1-3$ (Mac Low et al. 1998; Gammie 2001), and

$$t_{\rm SF} \equiv \frac{t_{\rm dyn}}{\epsilon_{\rm SF}} \,, \tag{21}$$

with $\epsilon_{\rm SF} \approx 0.02$ (Silk 1997; Genzel et al. 2010). With these fiducial values eq. (19) becomes

$$\frac{\sigma}{V_{\rm rot}} = \sqrt{\epsilon_{\rm SF} \gamma_{\rm dis}} \approx 0.2 \,. \tag{22}$$

In addition to the uncertainty in these parameters, several numerical factors of order unity have been omitted in this simple derivation, which will be recovered in Section 2.3. Despite the fact that the numerical values of $\epsilon_{\rm SF}$, $\gamma_{\rm dis}$ and the other parameters are not known to great accuracy, we learn from eq. (22) that the available power in the external accretion is in the same ballpark as the power required at $z \sim 2$ for maintaining the turbulence in the discs. As long as the conversion efficiency of that energy into turbulent energy is high, this could in principle be the main driver of disc turbulence. At low redshift, the cosmological accretion carries more than enough energy to maintain the observed $\sigma/V_{\rm rot} \approx 0.05$.

As an aside, the contribution of stellar feedback to driving turbulence in the disc can be estimated in a similar way, replacing the gravitational potential $V_{\rm in}^2$ by the energy provided by stars per unit stellar mass formed, $V_{\rm FB}^2$. Thus $\dot{E}_{\rm cosmo} \approx \dot{M}_{\rm cosmo} V_{\rm rot}^2$ is replaced by $\dot{E}_{\rm FB} = \dot{M}_{\rm SF} \cdot V_{\rm FB}^2$. With $\sim 10^{51}$ erg released by each supernova and one supernova per $100 \, {\rm M}_\odot$ of stars formed, the released energy corresponds to $V_{\rm FB} \sim 700 \, {\rm km s}^{-1}$, well above what is needed for driving the observed turbulence. However, the vast majority of the energy emitted is radiated away in the initial phases of the supernova evolution (Dekel & Silk 1986; Thornton et al. 1998), such that the energy available to be deposited in the ISM corresponds to only $V_{\rm FB} \sim 100 \, {\rm km s}^{-1}$, lower than the required energy. Furthermore, a large fraction of that energy is likely to drive outflows from the disc rather than turbulence inside the disc (Mac Low & Ferrara 1999; Joung et al. 2009; Ostriker & Shetty 2011).

2.2 Mass steady state

We now generalize the mass sink term in eq. (7) to include several additional processes, and have at steady state

$$\dot{M}_{\rm g} = \dot{M}_{\rm cosmo} - \dot{M}_{\rm SF} - \dot{M}_{\rm w,out} + \dot{M}_{\rm w,in} - \dot{M}_{\rm inf} = 0. \tag{23}$$

In the following, we describe the parameterization of the various sink terms. Table 1 gives an overview of all model parameters.

• The star-formation rate (SFR) is defined as

$$\dot{M}_{\rm SF} \equiv D \epsilon_{\rm SF} \frac{M_{\rm g}}{t_{\rm dyn}}, \tag{24}$$

where D is a duty cycle (with a fiducial value of 1, to be discussed in Section 4.1) and $\epsilon_{\rm SF}$ is the fraction of the gas that turns into stars every dynamical time $t_{\rm dyn}$ whenever star formation is 'on' ($D = 1$), as in eq. (21).

• It is assumed that stellar feedback blows galactic winds at a rate that is proportional to the SFR with a mass-loading factor η, namely

$$\dot{M}_{\rm w,out} \equiv \eta \cdot \dot{M}_{\rm SF}, \tag{25}$$

where typically $\eta \sim 1$. Galactic winds can also be driven by feedback from an Active Galactic Nucleus (AGN; e.g. Nesvadba et al. 2008), which, for simplicity, we include in the parameter η, since AGN activity and star formation are often concurrent.

• It is assumed that some fraction $\gamma_{\rm rcl}$ of the mass that was blown out into the wind comes back as an instantaneous fountain:

$$\dot{M}_{\rm w,in} \equiv \gamma_{\rm rcl} \cdot \eta \cdot \dot{M}_{\rm SF} \,. \tag{26}$$

The fiducial value we choose is $\gamma_{\rm rcl} = 0$, namely no returning winds.

• Disc instability is associated with torques that drive angular momentum out and mass in (Gammie 2001), to form a bulge at the disc centre. Part of this is clump migration, whose rate could be computed by dynamical friction or clump-clump interactions. The inflow rate can be expressed as

$$\dot{M}_{\rm inf} \equiv D \gamma_{\rm inf} \frac{M_{\rm g}}{t_{\rm dyn}} = \gamma_{\rm inf} \epsilon_{\rm SF}^{-1} \dot{M}_{\rm SF} \,, \tag{27}$$

where $\gamma_{\rm inf}$ is the inflow 'efficiency' per dynamical time (Dekel et al. 2009b, eqs. (19) and (24)). Based on eq. (5), we estimate $\gamma_{\rm inf} \approx 0.02$ at $z \sim 2$, and $\gamma_{\rm inf} \approx 0.001$ at $z \sim 0$. To avoid underestimating the effect of disc inflows, we use a fiducial value of $\gamma_{\rm inf} \epsilon_{\rm SF}^{-1} = 1$. The parameter D in eq. (27) is the same duty cycle as in eq. (24) for the SFR, assumed to be determined by the instability duty cycle.

By solving eq. (23), we obtain the steady-state solution

$$M_{\rm g} = \frac{\dot{M}_{\rm cosmo} t_{\rm dyn}}{D \epsilon_{\rm SF} [1 + \eta(1 - \gamma_{\rm rcl}) + \gamma_{\rm inf} \epsilon_{\rm SF}^{-1}]}. \tag{28}$$

The contribution of each term is easy to understand qualitatively. The gas mass is proportional to the cosmological supply rate of gas. More vigorous outflows (large η) reduce the gas mass in the disc, unless they are largely recycled (large $\gamma_{\rm rcl}$). A duty cycle with longer 'off' phases (small D) leaves more gas in the disc. A higher efficiency of star formation (large $\epsilon_{\rm SF}$) reduces the gas mass. A stronger inflow inside the disc (large $\gamma_{\rm inf}$) also reduces the disc gas mass. Using eq. (24), we also obtain

$$\dot{M}_{\rm SF} = \frac{\dot{M}_{\rm cosmo}}{1 + \eta(1 - \gamma_{\rm rcl}) + \gamma_{\rm inf} \epsilon_{\rm SF}^{-1}}. \tag{29}$$

Table 1. Model parameters.

Parameter	Fiducial Value	Definition	Equation
ϵ_{SF}	0.02	Star-formation efficiency per disc dynamical time	(24)
D	1	Duty cycle for instability, star-formation and inflows	(24),(27)
η	1	Wind mass-loading factor	(25)
γ_{rcl}	0	Fraction of instantaneously recycled wind	(26)
γ_{inf}	0.02	Fraction of gas inflowing inside the disc per dynamical time	(27)
γ_{dis}	2	Ratio of the turbulence dissipation timescale to the dynamical time	(20)
ξ_i	1	Fraction of the in-streaming kinetic energy that turns into turbulence	(30)
ξ_m	0	Fraction of the disc inflow potential energy that turns into turbulence	(30)
u	$1/\sqrt{2}$	Ratio of V_{rot} to the in-streaming velocity	(31)
$\nu - 1$	0	The slope of the logarithmic rotation curve, $(\log V_{rot}/\log r)$	(3)

2.3 Turbulent Energy steady state

Gravitational energy can be transferred into turbulent energy to compensate for the dissipative losses in two general ways. If a large fraction of the incoming streams that hit the disc is in dense gas clumps, they can transfer momentum into the disc gas, and thus convert the stream kinetic energy into turbulence (Genzel et al. 2008; Dekel et al. 2009b; Elmegreen & Burkert 2010). Alternatively, the rotational and potential energy of the disc mass becomes available due to the instability-driven mass inflow within the disc, including clump migration, where turbulence is generated by the same torques that are responsible for angular-momentum outflow and the associated mass inflow (Dekel et al. 2009b; Krumholz & Burkert 2010; Cacciato et al. 2011; Forbes et al. 2011). The quasi-steady state of the turbulent energy in the disc is then described by

$$
\begin{aligned}
\dot{K}_g & = \xi_i(\dot{M}_{cosmo} + \dot{M}_{w,in})0.5V_{in}^2 + \xi_m\dot{M}_{inf}V_{rot}^2 \\
& - (\dot{M}_{SF} + \dot{M}_{w,out} + \dot{M}_{inf})1.5\sigma^2 - M_g1.5\sigma^2 t_{dis}^{-1} = 0,
\end{aligned}
\tag{30}
$$

where the different terms are as follows:

• It is assumed that both the cosmological accretion and the recycled wind arrive to the disc at a speed

$$
V_{in} \equiv V_{rot}/u,
\tag{31}
$$

and that a fraction ξ_i of the kinetic energy they carry is converted into disc turbulent energy. We take $u^2 = 0.5$ (see Appendix C), and a fiducial maximum value $\xi_i = 1$, so that we can examine the maximum possible contribution of in-streaming energy.

• It is assumed that as gas flows in or migrates to the galaxy centre, potential energy, which is released at a rate of $\sim \dot{M}_{inf}V_{rot}^2$, is transformed into turbulent energy with an efficiency ξ_m. See Appendix B for a discussion of this assumption. We later examine both the effects of $\xi_m = 0$ and $\xi_m = 1$.

• The gas sink terms, i.e. star formation, outflowing winds and inflows to the bulge, take their share of turbulent energy when they leave the disc.

• The turbulent energy dissipates on a dissipation timescale $t_{dis} \equiv \gamma_{dis}t_{dyn}$, as in eq. (20), with a fiducial value $\gamma_{dis} = 2$.

We solve eq. (30) for σ/V_{rot}, using eqs. (28) and (29), and obtain

$$
\frac{\sigma^2}{V_{rot}^2} = \frac{2/3}{u^2/0.5}\frac{\xi_i(1 + \eta + \gamma_{inf}\epsilon_{SF}^{-1}) + \xi_m\gamma_{inf}\epsilon_{SF}^{-1}(u^2/0.5)}{(\epsilon_{SF}\gamma_{dis}D)^{-1} + (1 + \eta + \gamma_{inf}\epsilon_{SF}^{-1})}.
\tag{32}
$$

In the limit[2] $1 + \eta + \gamma_{inf}\epsilon_{SF}^{-1} \ll (\epsilon_{SF}\gamma_{dis}D)^{-1}$ and with $u^2 = 0.5$, we obtain

$$
\frac{\sigma}{V_{rot}} \approx 0.16\sqrt{\epsilon_{SF,0.02}\gamma_{dis,2}D}
\tag{33}
$$

$$
\times \sqrt{\xi_i(1 + \eta + \gamma_{inf}\epsilon_{SF}^{-1}) + \xi_m\gamma_{inf}\epsilon_{SF}^{-1}},
$$

where $\epsilon_{SF,0.02} \equiv \epsilon_{SF}/0.02$ and $\gamma_{dis,2} \equiv \gamma_{dis}/2$.

As already noted for the approximate steady-state solution in eq. (22), the more detailed result for σ/V_{rot} in eq. (32) is also independent of $\dot{M}_{cosmo}$, because in our current model the latter controls both the incoming energy and the disc gas mass that is involved in the turbulence, so the varying cosmological supply always provides the same *specific* turbulent energy.

3 CASE I: NON-REGULATED DISC INSTABILITY

3.1 The relative roles of in-streaming and disc inflow

The approximate expression $\dot{E}_{cosmo} \approx \dot{M}_{cosmo}V_{rot}^2$ used for the simple derivation in Section 2.1 does not distinguish between cosmological in-streaming kinetic energy and gravitational potential energy released during inflows inside the disc. Each of these components carries similar specific energy of $\approx V_{rot}^2$ (Appendix C). Inspection of eq. (33) elucidates their individual roles.

The conversion of rotational and potential energy into turbulent energy depends on inflow in the disc, i.e. ξ_m couples to γ_{inf}. The ability of the kinetic energy of in-streaming mass to be converted into turbulence in the disc depends on complex physical processes in the vicinity of the disc, which

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[2] With the fiducial values (Table 1), $1 + \eta + \gamma_{inf}\epsilon_{SF}^{-1} = 3$ and $(\epsilon_{SF}\gamma_{dis}D)^{-1} = 25$, so we consider the approximation $1 + \eta + \gamma_{inf}\epsilon_{SF}^{-1} \ll (\epsilon_{SF}\gamma_{dis}D)^{-1}$ a good one. More generally, η and $\gamma_{inf}\epsilon_{SF}^{-1}$ are not expected to have a value of more than a few, while $(\epsilon_{SF}\gamma_{dis}D)^{-1}$ is expected to have a value of several tens.

we simply parameterize with ξ_i. If $\xi_i > 0$, the accretion energy acts as a direct external driver of turbulence independently of disc inflows or winds. However, the presence of disc inflows and/or winds enhances the contribution of in-streaming in driving turbulence, i.e. ξ_i couples both to γ_{inf} and η. Thus, the relative contribution of kinetic and potential energy of cosmological origin depends not only on the intrinsic efficiencies ξ_i and ξ_m, but also on the importance of winds and disc inflows.

3.2 The effects of galactic winds and disc inflow

By comparing eqs. (28) and (15) we can verify the role played by winds and inflows in the disc on the gas mass and hence the SFR. The gas mass and SFR are suppressed by escaping winds. In the limit of very strong winds and little recycling, $\eta(1-\gamma_{\mathrm{rcl}}) \gg 1+\gamma_{\mathrm{inf}}\epsilon_{\mathrm{SF}}^{-1}$, the gas mass and SFR are roughly proportional to η. If the winds are fully recycled, $\gamma_{\mathrm{rcl}} = 1$, then they have no net effect on the gas mass and SFR, given that recycling is assumed to be instantaneous.

On the other hand, the value of σ is independent of the recycling rate γ_{rcl}. This is despite the fact that the origin of the dependence on η does depend on γ_{rcl}. For example, If there is no recycling, $\gamma_{\mathrm{rcl}} \approx 0$, σ is higher for larger η because the gas mass is smaller while the energy input is the same. On the other hand, if the outflows are all recycled back to the galaxy, $\gamma_{\mathrm{rcl}} \approx 1$, σ is higher for larger η because the returning winds add to the energy provided by the cosmological accretion while the gas mass remains unchanged. We note that, regardless of the level of recycling, stronger outflows drive an increase in σ, despite the fact that we have ignored direct local deposit of feedback energy in the disc gas. This is done indirectly, either by lowering the gas mass or by adding to the energy brought by external accretion.

The inflow in the disc depletes the gas mass and thus suppresses the SFR, via $\gamma_{\mathrm{inf}}\epsilon_{\mathrm{SF}}^{-1}$, a quantity of order unity. The turbulent velocity σ is even more sensitive to the inflow in the disc because (a) when the gas mass is depleted the same amount of energy input by in-streaming results in higher σ (via ξ_i), and (b) the inflow down the potential gradient in the disc contributes energy to driving σ up (via ξ_m).

Can the observed velocity dispersion at $z \sim 2$ be primarily driven by the cosmological in-streaming? Recall that the observed values are typically $\sigma/V_{\mathrm{rot}} \sim 0.2$, perhaps even ~ 0.3. If the conversion efficiency of in-streaming energy to turbulence is high, $\xi_i \approx 1$, and the SFR and dissipation rate are at their fiducial values, the term referring to the streams by themselves already provides $\sigma/V_{\mathrm{rot}} \approx 0.16$, which is in the ballpark of the desired value, though slightly short. With the fiducial depletion by winds, $\eta \sim 1$, this becomes $\sigma/V_{\mathrm{rot}} \gtrsim 0.2$. With the fiducial disc inflow, $\gamma_{\mathrm{inf}}\epsilon_{\mathrm{SF}}^{-1} \sim 1$ (and $\xi_m \approx 1$), even without winds, it becomes $\sigma/V_{\mathrm{rot}} \lesssim 0.3$. Adding the fiducial winds and disc inflow, we obtain $\sigma/V_{\mathrm{rot}} \gtrsim 0.3$. We conclude that if somehow $\xi_i \sim 1$, the gravitational energy associated with the inflow, and in particular the clumpy gas streaming into the disc, can have a significant contribution to the disc turbulence, which can be naturally aided by outflow depletion and disc inflow.

3.3 Evolution of Q

If the values of the parameters D, ξ_i, and ξ_m are fixed, both M_{g} and σ are determined by a balance between the externally set in-streaming, the star formation, the winds, and the inflows in the disc. In this case, the Toomre Q parameter is not necessarily locked to $Q \sim 1$. Self-regulation of the instability at $Q \sim 1$ requires that the relevant physical processes, such as the inflow rate in the disc, the SFR and the outflow rate, adjust themselves to maintain $Q \sim 1$, and this case, where the relevant parameters are not fixed, is deferred to Section 4.

The Toomre stability parameter is

$$Q = \frac{\sigma\kappa}{\pi G \Sigma_{\mathrm{g}}}, \tag{34}$$

where $\kappa = \sqrt{2}\nu\Omega$. Using $\Omega = V_{\mathrm{rot}}/R$, $V_{\mathrm{rot}}^2 = GM_{\mathrm{tot}}/R$, $M_{\mathrm{tot}} = M_{\mathrm{g}}/\delta$, and $M_{\mathrm{g}} = \pi R^2 \Sigma_{\mathrm{g}}$, we obtain

$$Q = (2\nu)^{1/2}\delta^{-1}\frac{\sigma}{V_{\mathrm{rot}}}. \tag{35}$$

Then using the approximate solution in eq. (33) for σ/V_{rot}, and assuming a flat rotation curve $\nu = 1$, we obtain

$$\begin{aligned} Q &\approx 0.68\delta_{0.33}^{-1}\sqrt{\epsilon_{\mathrm{SF},0.02}\gamma_{\mathrm{dis},2}D} \\ &\times \sqrt{\xi_i(1+\eta+\gamma_{\mathrm{inf}}\epsilon_{\mathrm{SF}}^{-1})+\xi_m\gamma_{\mathrm{inf}}\epsilon_{\mathrm{SF}}^{-1}}, \end{aligned} \tag{36}$$

where the disc mass fraction (eq. (2)) is expressed by $\delta_{0.33} \equiv \delta/0.33$. Substituting the fiducial values from Table 1, we obtain $Q \approx 1.18\delta_{0.33}^{-1}$.

From eq. (35), in the non-self-regulated case studied here, where σ/V_{rot} is constant, we learn that Q scales with mass and time as $Q \propto \delta^{-1}$, where $\delta = M_{\mathrm{g}}/M_{\mathrm{tot}}$ within the disc radius. We can evaluate the time evolution of Q through δ as follows. From eq. (28), the steady-state solution is $M_{\mathrm{g}} \propto \dot{M}_{\mathrm{cosmo}}t_{\mathrm{dyn}}$. Assuming that the disc radius is proportional to the halo virial radius, $R = \lambda R_{\mathrm{vir}}$, with λ a constant spin parameter[3], we obtain $t_{\mathrm{dyn}} \propto t_{\mathrm{Hubble}}$, which in the Einstein-deSitter phase that is approximately valid at $z > 1$ gives

$$t_{\mathrm{dyn}} \propto (1+z)^{-3/2}. \tag{37}$$

Based on the EPS approximation (confirmed by fits to cosmological simulations), the cosmological input rate can be approximated in the Einstein-deSitter phase by

$$\dot{M}_{\mathrm{cosmo}} \propto M_{\mathrm{vir}}(1+z)^{5/2} \tag{38}$$

(Neistein et al. 2006; Neistein & Dekel 2008). Assuming that the halo mass profile is roughly $M(r) \propto r$, we have

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[3] This is based on the assumption of gas angular momentum conservation during the collapse to the disk (Mo et al. 1998). While numerical studies have suggested that this picture neglects many details (e.g. Zavala et al. 2008; Dutton & van den Bosch 2009; Danovich et al. 2012; Aumer & White 2012), it provides a fair match to observed disk sizes (e.g. Mo et al. 1998; Bullock et al. 2001; Maller & Dekel 2002; Dutton et al. 2007; Burkert et al. 2010; Dutton & van den Bosch 2012).

$M_{\rm tot} \simeq \lambda M_{\rm vir}$ [4]. The above finally yield for the non-self-regulated model

$$Q \propto \delta^{-1} \propto (1+z)^{-1} . \tag{39}$$

We learn from eqs. (36) and (39) that the discs tend to be unstable at high redshifts, $Q < 1$, and then evolve toward stabilisation at later times, $Q > 1$. We can see that the growth of Q in time is driven by the decline of the cosmological accretion rate, eq. (38), being steeper than the increase in time of $t_{\rm dyn}$. This trend is in at least qualitative agreement with observations that find Q to be of order a few in local disc galaxies (e.g. Leroy et al. 2008; Watson et al. 2012), but around, or even below, unity at high redshift (Genzel et al. 2011).

Eq. (36) shows that at high redshift, violent disc instability with $Q \lesssim 1$ is naturally driven by the high δ, which results from the intense in-streaming rate, eq. (28). The in-streaming power, even when the efficiency for driving turbulence is high, $\xi_i = 1$, may not be able to drive turbulence with $\sigma/V_{\rm rot}$ high enough for balancing the high δ and thus stabilising the disc. The instability is associated with disc inflow of a large $\gamma_{\rm inf}$, which can drive further turbulence both by providing energy that is converted to turbulence if ξ_m is high and by depleting the disc mass (see Section 4.3). High-redshift observations reveal unstable discs where $Q \lesssim 1$, with a high δ (Tacconi et al. 2010) as well as a high η (e.g. Steidel et al. 2010; Genzel et al. 2011). The latter may indicate that feedback also plays a role in maintaining $Q \sim 1$, either indirectly by depletion of the disc gas through outflows, or by direct injection of feedback energy into turbulence (not considered here). Some combination of disc inflows and feedback/outflows is capable of providing turbulence at the level that would stabilize the disc, and this may occur in a self-regulated way that keeps the disc marginally unstable. At low redshift, where $\delta \ll 1$, Q could be higher than unity even without the presence of winds and disc inflows (see also our complementary calculation in Cacciato et al. 2011).

We can replace the assumption of a constant loading factor η with a prediction for momentum-driven winds, $\eta \propto V^{-1}$ (Murray et al. 2005). Using the standard cosmological virial relation this implies

$$\eta \propto M^{-1/3}(1+z)^{-1/2} . \tag{40}$$

Plugging this in eq. (33), in the limit of strong outflows $\eta \gg 1$, we obtain

$$\frac{\sigma}{V_{\rm rot}} \propto \eta^{1/2} \propto V_{\rm rot}^{-1/2} \propto M^{-1/6}(1+z)^{-1/4} . \tag{41}$$

Then from eq. (35), using eq. (28) that implies $\delta \propto \eta^{-1}(1+z)$, we obtain[5]

$$Q \propto M^{-1/2}(1+z)^{-7/4} . \tag{42}$$

We learn that the variation of η according to momentum-driven winds enhances the redshift dependence of Q compared to the constant η case, eq. (39). Unfortunately, the slight growth of $\sigma/V_{\rm rot}$ in time in eq. (41) makes the agreement with the observed *decline* of $\sigma/V_{\rm rot}$ even worse than in the constant η case. The consideration of momentum-driven winds also introduces a mass dependence. The predicted weak mass dependence of $\sigma/V_{\rm rot} \propto (1+z)^{-1/4}$ seems reasonable, although the observational trend is not yet well established. For example, Klessen & Hennebelle (2010) find that $\sigma/V_{\rm rot} \propto M^{-1/3}$ in the local Universe, while van der Kruit & Freeman (2011) report $\sigma/V_{\rm rot} \approx$ const. At $z \sim 2$, the largest existing observational sample shows no clear trend, though still with significant scatter that makes the situation inconclusive (Mancini et al. 2011).

The observed mass dependence of Q in galaxy discs is not yet well established. While Dalcanton et al. (2004) find a sharp threshold for the onset of instability at $V_{\rm rot} > 120\,{\rm km s}^{-1}$, Watson et al. (2012) find no trend of Q with galaxy mass. Our predicted mass dependence of Q depends on the scaling of δ and the different model parameters with mass. In the case of constant model parameters, we obtain that Q is independent of mass, eq. (39), and in the case of momentum-driven winds scaling, we obtain that more massive discs are less stable, eq. (42).

4 CASE II: SELF-REGULATED DISC INSTABILITY

In Section 3 we have shown that case I produces gas discs in which $\sigma/V_{\rm rot}$ is constant in time and Q is gradually increasing to values larger than unity. While the systematic increase in Q toward low redshifts may be consistent with the observational trend toward a larger abundance of stable discs, the constancy of $\sigma/V_{\rm rot}$ is clearly in conflict with the observed decline of this quantity. In this section, we appeal to an alternative case II, where self-regulation at $Q \sim 1$ is imposed at all times. We address the possibility that the self-regulation is achieved by periodic episodes where the instability and the associated SFR and disc inflows are 'on' or 'off' with a duty cycle $D < 1$ (see also Martig et al. 2009). Self-regulation may alternatively be achieved by adjustments of the disc inflow rate, via $\gamma_{\rm inf}$ and ξ_m, as in our companion paper (Cacciato et al. 2011). We also check the effect of a systematic decline with time of the accretion conversion efficiency ξ_i, perhaps reflecting the evolution from dense narrow streams to a wide-angle accretion and a gradual decrease in the gas fraction and degree of clumpiness in the accreting gas.

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[4] This is formally exact for an isothermal sphere, which is a reasonable approximation for the dark matter halo. Since typically $\lambda \sim 0.03$ (Bullock et al. 2001), and the baryons in the central galaxy are known to be less massive than $\approx 0.03 M_{\rm vir}$ (Guo et al. 2010; Behroozi et al. 2010), we find that this is a reasonable approximation altogether.

[5] Note that there could be tension between the limit $1 + \eta + \gamma_{\rm inf}\epsilon_{\rm SF}^{-1} \ll (\epsilon_{\rm SF}\gamma_{\rm dis}D)^{-1}$ in which eq. (33) is valid, and the limit $\eta \gg 1$ in which eq. (41) is valid. They may both be valid if $\eta \sim 5$, say. If η is larger, we can relax the first condition and use eq. (32)

instead of eq. (33). We then find that the scaling of Q in the $\eta \gg 1$ limit is bounded by the scalings in eqs. (39) and (42), and the scaling of $\sigma/V_{\rm rot}$ is similarly bounded by the flat value of eq. (32) and the weak variation in eq. (41).

4.1 Maximum conversion efficiencies and a duty cycle

We assume here that once Q rises to slightly above unity, the disc tends toward stabilisation, star formation is suppressed (Kennicutt 1989; Martin & Kennicutt 2001), and the galactic outflow as well as inflow within the disc become weaker too. This allows Σ_g to increase in response to the continuing cosmological accretion, while the growth of σ slows down. As a result, Q tends to decrease to slightly below unity, back to an unstable phase where star formation, outflows and disc inflow resume, and so on. We thus expect oscillations about $Q \sim 1$. We model this by a duty cycle $D < 1$ that represents the fraction of time when the instability is 'on'. Here we adopt maximum efficiencies for energy conversion to turbulence, $\xi_i = \xi_m = 1$. From eq. (36), with $Q = 1$, we obtain

$$D \simeq \frac{2.2\delta_{0.33}^2}{\epsilon_{\mathrm{SF},0.02}\gamma_{\mathrm{dis},2}(1 + \eta + 2\gamma_{\mathrm{inf}}\epsilon_{\mathrm{SF}}^{-1})} . \tag{43}$$

With the fiducial parameter values (Table 1), and a high cold mass fraction characteristic of high-redshift discs, $\delta \simeq 0.33$, we have $D \lesssim 1$. When the gas fraction is even higher, the outflow rate is lower, or the disc inflow rate is lower, eq. (43) may give $D > 1$, which is clearly unphysical. This is another representation of the result from Section 3.3 that at high redshift, when δ is high, in the absence of winds and when the disc inflow is ignored, there is hardly enough power in the cosmological in-streaming by itself to drive the high turbulence required for $Q = 1$. The fiducial winds, $\eta \sim 1$, or the fiducial disc inflow, $\gamma_{\mathrm{inf}}\epsilon_{\mathrm{SF}}^{-1} \sim 1$, help obtaining a physical result with $D < 1$. Another way to obtain $D < 1$ may be if during the violent instability phase the timescales for star formation and inflow somehow become shorter than the disc dynamical time t_{dyn}, while the dissipation timescale is long compared to t_{dyn}.

On the other hand, at low redshift, with $\delta \lesssim 0.1$, and even more so if γ_{inf} and η are non-negligible, the resulting duty cycle D drops significantly below unity to $\lesssim 0.1$ in order to keep $Q = 1$. From eq. (43), the scaling of D with mass and redshift is $D \propto M^0(1 + z)^{2/3}$ in the limiting case of weak winds ($\eta \ll 1$), and $D \propto M^{1/3}(1+z)^{7/6}$ in the limit of strong momentum-driven winds with no recycling ($\eta \gg 1$ and $\gamma_{\mathrm{rcl}} = 0$). In the general case between these limits, D is declining with time close to linearly with $(1 + z)$, such that at later times one expects to detect a smaller fraction of the galaxies in the unstable phase.

When forcing $Q = 1$ and keeping D free, from eqs. (28) and (33) and the definitions for κ and Σ_g, we obtain

$$\Sigma_g = \frac{\kappa}{\pi}\left(\frac{\gamma_{\mathrm{dis}}\dot{M}_{\mathrm{cosmo}}(1 + \eta + 2\gamma_{\mathrm{inf}}\epsilon_{\mathrm{SF}}^{-1})}{3\sqrt{2}u^2G^2(1 + \eta(1 - \gamma_{\mathrm{rcl}}) + \gamma_{\mathrm{inf}}\epsilon_{\mathrm{SF}}^{-1})}\right)^{\frac{1}{3}} \tag{44}$$

$$\approx 36\,\mathrm{M}_\odot\,\mathrm{pc}^{-2}\left(\frac{35\,\mathrm{Myr}}{\kappa^{-1}}\right)\left(\frac{\dot{M}_{\mathrm{cosmo}}}{1\,\mathrm{M}_\odot\,\mathrm{yr}^{-1}}\right)^{\frac{1}{3}}$$

and

$$\sigma = \left(\frac{G\dot{M}_{\mathrm{cosmo}}\gamma_{\mathrm{dis}}(1 + \eta + 2\gamma_{\mathrm{inf}}\epsilon_{\mathrm{SF}}^{-1})}{3\sqrt{2}u^2(1 + \eta(1 - \gamma_{\mathrm{rcl}}) + \gamma_{\mathrm{inf}}\epsilon_{\mathrm{SF}}^{-1})}\right)^{\frac{1}{3}} \tag{45}$$

$$\approx 17.5\,\mathrm{km s}^{-1}\left(\frac{\dot{M}_{\mathrm{cosmo}}}{1\,\mathrm{M}_\odot\,\mathrm{yr}^{-1}}\right)^{\frac{1}{3}},$$

where the second equality in each of these equations is calculated using our fiducial parameters from Table 1. In contrast with the solution in Section 3, here σ/V_{rot} does depend on the external cosmological in-streaming rate $\dot{M}_{\mathrm{cosmo}}$. This is a result of the imposed self-regulation, where a decrease in $\dot{M}_{\mathrm{cosmo}}$ induces similar decreases in σ/V_{rot} and in δ such that Q remains constant, eq. (35). These evolution trends agree with the observed trends better than those predicted in case I. In particular, the results obtained for $\dot{M}_{\mathrm{cosmo}} = 1\,\mathrm{M}_\odot\,\mathrm{yr}^{-1}$ and $\kappa = (35\,\mathrm{Myr})^{-1}$, as roughly appropriate for the Milky Way, are close to the typical values in local disc galaxies, and if $\dot{M}_{\mathrm{cosmo}}$ is scaled up to $\approx 100\,\mathrm{M}_\odot\,\mathrm{yr}^{-1}$, as appropriate for high redshift discs, one obtains $\Sigma_g \approx 200\,\mathrm{M}_\odot\,\mathrm{pc}^{-2}$ and $\sigma \approx 80\,\mathrm{km s}^{-1}$, which are indeed in the observed ballpark (Förster Schreiber et al. 2009; Genzel et al. 2010). In case II the gas mass M_g declines at a slower rate, because the overall gas depletion by star formation, outflows and disc inflow is slower in accord with the low duty cycle D. An examination of eq. (28) using the aforementioned scaling of D in this solution, as well as eqs. (37) and (38), gives $M_g \propto M(1+z)^{1/3}$. Similarly, eqs. (44) and (45) give $\Sigma_g \propto M^{1/3}(1 + z)^{7/3}$ and $\sigma \propto M^{1/3}(1+z)^{5/6}$, respectively, and $\sigma/V_{\mathrm{rot}} \propto M^0(1+z)^{1/3}$, for any constant η. These trends with redshift agree reasonably well with observations as well. The associated evolution rate of the gas fraction, $M_g/M \propto (1+z)^{1/3}$, is however somewhat weaker than observations indicate for massive galaxies between $z \approx 2$ and $z = 0$ (see Bouché et al. (2010) and references therein).

We note, however, that a decreasing star formation duty cycle, in particular one that decreases as strongly as δ^2, should manifest itself in observations. If the 'off' periods are long compared to the galactic dynamical time, $D \ll 1$ will result in a large fraction ($\approx 1 - D$) of disc galaxies having significant gas discs with no star-formation. There are indeed several examples of observed gas discs that have low levels of star-formation, possibly as a result of their stability (e.g. Martig et al. 2009; MacLachlan et al. 2011). However, for typical disc galaxies this is not the case. Indeed, Leroy et al. (2008) and Watson et al. (2012) find no clear correlation between star-formation efficiency and Q. On the other hand, if the 'off' periods are short compared to the galactic dynamical time, they will result in discs being in a mixed state of 'on' and 'off' in different regions on the disc. In a statistical sense, this will show in observations as a decrease of the mean star-formation efficiency with cosmic time. Such a trend may be observed, but probably not to the extent that is predicted by our model (Genzel et al. 2010; Daddi et al. 2010; Krumholz et al. 2012). Disc galaxies are observed to have, on average, similar star-formation efficiencies per dynamical time at low and high redshift. This remains an open problem with regards to this scenario.

4.2 Sub-maximal efficiencies and no duty cycle

We now allow only a fraction ξ_i of the incoming kinetic energy to be converted into turbulent energy, and solve for the value of ξ_i that is required to keep $Q = 1$. We start with the accretion energy as the only direct driver of turbulence, namely $\xi_m = 0$, and assume $D = 1$. From eq. (36), with $Q = 1$, we obtain

$$\xi_i \simeq \frac{2.2\delta_{0.33}^2}{\epsilon_{\mathrm{SF},0.02}\gamma_{\mathrm{dis},2}(1+\eta+\gamma_{\mathrm{inf}}\epsilon_{\mathrm{SF}}^{-1})}\ . \tag{46}$$

This is a very similar expression to the one found for D in eq. (43). However, the physical situation is different in the two cases. We note that δ evolves differently because in the $D \neq 1$ case M_{g} is larger than in the current case, as shown in Section 4.1. In the current solution, Σ_{g} is the same as in case I, i.e. it is decreasing in proportion to $\dot{M}_{\mathrm{cosmo}}$, but due to the requirement $Q = 1$, here σ is forced to decrease as well. This is achieved by a decreasing conversion efficiency ξ_i from in-streaming to turbulence.

The alternative, more physically motivated assumption of efficient conversion of the energy associated with the disc inflow, $\xi_m = 1$ (Appendix B), leads to

$$\xi_i \simeq \frac{2.2\delta_{0.33}^2\gamma_{\mathrm{dis},2}^{-1} - 50\gamma_{\mathrm{inf}}}{\epsilon_{\mathrm{SF},0.02}(1+\eta+\gamma_{\mathrm{inf}}\epsilon_{\mathrm{SF}}^{-1})}\ . \tag{47}$$

In both of these $\xi_i < 1$ cases, the scalings of M_{g}, δ and Σ_{g} with mass and time are the same as in case I. From $Q = 1$ we obtain $\sigma/V_{\mathrm{rot}} = \delta/\sqrt{2\nu}$, which implies

$$\sigma/V_{\mathrm{rot}} \propto M^0(1+z)^1 \tag{48}$$

without winds, and

$$\sigma/V_{\mathrm{rot}} \propto M^{-\frac{1}{3}}(1+z)^{1.5} \tag{49}$$

with escaping strong winds. The evolution of σ/V_{rot} in the cases where $\xi_i < 1$, with or without winds, agrees better with observations.

At lower redshift, as the cold disc fraction δ declines, the condition $Q = 1$ with $D = 1$ forces the conversion efficiency of the in-streaming to turbulence ξ_i to decline as well, $\xi_i \propto \delta^2$, as in eqs. (46) and (47). This means $\xi_i \propto (1+z)^2$ without winds and $\xi_i \propto M(1+z)^{3.5}$ with escaping strong winds. There could be several reasons for such a decline of ξ_i in time. First, the conversion factor could be a growing function of the gas fraction in the disc, namely of δ. Second, it is likely to be growing with the gas fraction in the accreting baryons, which is declining with time. Third, the evolution of the input pattern from narrow, dense streams at high redshift to a wide-angle accretion at late times makes the penetration of cold gas into the disc less efficient at later times (Dekel & Birnboim 2006; van de Voort et al. 2011). The lower gas density contrast between the streams and the disc is another reason for a smaller ξ_i, especially if it is somehow associated with a lower degree of clumpiness in the streams, as the coupling between in-streaming and disc arises from in-streaming gas of comparable density to the disc gas (Dekel et al. 2009b). However, except for the first reason that relates to the gas fraction in the disc, the value of ξ_i is determined externally, independently of the instability state of the disc or its other properties. Therefore a self-regulation loop is not expected to drive the required $\xi_i \propto \delta^2$ for $Q \sim 1$. In this case, it seems that the evolution of ξ_i can only match the requirements for $Q \sim 1$ by some coincidence.

4.3 Self-regulated inflow in the disc

Assuming fixed values for ξ_i, ξ_m and D, the inflow rate within the disc may adjust itself to compensate for the dissipative losses and maintain the instability at $Q \sim 1$.

This is the basis of the analysis in Krumholz & Burkert (2010), Forbes et al. (2011), and our companion paper Cacciato et al. (2011). With our fiducial choice of parameters, including outflows, this can be achieved quite naturally. In the absence of outflows, the disc inflow rate should be comparable to or somewhat higher than the SFR, $\gamma_{\mathrm{inf}} \gtrsim \epsilon_{\mathrm{SF}}$, depending on the exact values of the parameters ξ_i, ξ_m ,γ_{dis} and ϵ_{SF} (see a discussion of Krumholz & Burkert (2010) in Section 5). We note that an enhanced instability that boosts up the disc inflow rate would also enhance the SFR and outflow rate, which would help the self-regulation. However, if such a case is also accompanied by faster dissipation, then increased inflows may not be able to solve the problem. The details of the inflow within the disc, including gas and stars, clumps and off-clump material, is being investigated via cosmological hydrodynamical simulations (Cacciato et al., in preparation). This has additional important consequences regarding the issues of bulge growth and feeding central black holes (Bournaud et al. 2011a,b).

5 DISCUSSION: COMPARISON WITH THE LITERATURE

Several earlier studies evaluated the possible role of in-streaming and disc inflows in driving the observed velocity dispersion in $z \sim 2$ discs, reaching seemingly conflicting conclusions. The first estimates of this kind (Förster Schreiber et al. 2006; Genzel et al. 2008) suggested that it is plausible that there is enough energy in the cosmological in-streaming, depending on the exact numerical values of several parameters in their equations. In Dekel et al. (2009b) the in-streaming energy explicitly depends on the unknown small-scale clumpiness of the streams, and could therefore go either way. In Cacciato et al. (2011) we obtain $\sigma/V_{\mathrm{rot}} \approx 0.2$ when choosing favourable values for the relevant parameters. Klessen & Hennebelle (2010) estimated that there is enough energy in the cosmological accretion, and Krumholz & Burkert (2010) reached a similar conclusion when examining the gravitational potential energy that is released during mass inflows inside the discs. Khochfar & Silk (2009) concluded that only 18% of the in-streaming energy is required to reproduce the observed values. In apparent contrast, Lehnert et al. (2009) and Elmegreen & Burkert (2010) conclude that there is not enough in-streaming energy to account for $z \sim 2$ turbulence.

As we show in Section 2, the value of σ/V_{rot} depends basically on two timescales, one associated with the decay of turbulence (parameterized with γ_{dis}), and the other associated with the gas mass conservation in the disc (related to ϵ_{SF}, η and γ_{inf}). All of the aforementioned studies involved a turbulent energy balance that is very similar to the one we consider in this work. However, they differ from one another in the choices of the turbulent dissipation timescale. Moreover, some of these models do not explicitly include a mass steady state condition, and differ in the *implicit* assumptions they make regarding the timescale associated with the mass equation. A closer inspection of the different assumptions made, which we perform next, reveals how they lead to the apparently conflicting conclusions.

Genzel et al. (2008) do not explicitly write a steady state equation for the gas mass conservation, but their treat-

ment is equivalent to the simple equations in Section 2, only with t_{SF} crudely approximated by the specific rate of cosmological accretion t_{acc} (from the growth of dark matter haloes), hence not addressing the possible difference between the rates of star formation and accretion. They remain agnostic as for whether there is enough in-streaming energy for driving the observed $\sigma/V_{rot} \approx 0.2-0.3$, due to uncertainties in numerical values of order unity. Nevertheless, their fiducial values[6] indicate a small value of $\sigma/V_{rot} \approx 0.07$, implying that an additional source of energy is required. Apart from the choice of parameter values, there is a significant difference between our models. As t_{acc} declines with time faster than the dynamical time, Genzel et al. (2008) naturally obtain that σ/V_{rot} decreases with time (see eq. (19)). In our model, M_g follows $\dot{M}_{cosmo}$ (eq. (28)) and becomes smaller with time, so that the effective timescale for specific mass and energy gain is constant (and related to t_{dyn}). With the Genzel et al. (2008) implicit assumption $t_{SF} = t_{acc}$, the gas mass remains constant with redshift, since the star-formation timescale becomes longer at the same rate that the accretion diminishes.

Khochfar & Silk (2009) find that a conversion efficiency of only $\xi_i = 18\%$ is required to obtain high σ/V_{rot} as observed. Their semi-analytical model does not explicitly include a steady-state solution to the gas disc mass as we do here, but gas velocity dispersions are obtained in their model as a result of similar physical considerations to the ones made here in Section 2.1. Therefore, it is straightforward to compare the two analyses and find that they require a lower conversion efficiency ξ_i as a result of the higher star-formation efficiency ϵ_{SF} (α in their notation) they assume for their porosity-driven star-formation model, $\epsilon_{SF} \approx 0.02(\sigma/10\,\mathrm{km\,s^{-1}}) \approx 0.15$ at $z \sim 2$.

The set of equations developed in Elmegreen & Burkert (2010) is very similar to our equations here. They do not consider the winds (η) and disc inflow (γ_{inf}) terms, while they do consider a constant conversion efficiency ξ_i (ϵ in their notation) from in-streaming kinetic energy to turbulent energy and no star formation when $Q > 1$. Elmegreen & Burkert (2010) focus on the transient state before the quasi-steady state, an issue we do not address here. Nevertheless, they obtain that once the quasi-steady state is reached, the accretion cannot be the dominant source of turbulence, because it cannot provide enough turbulence to keep the disc at $Q \sim 1$. When we neglect winds and disc inflows, we arrive at a similar conclusion, unless we choose favourable parameter values such as $\epsilon_{SF} = 0.03$ and $\gamma_{dis} = 3$, for which we obtain $\sigma/V_{rot} = 0.24$ (see eq. (33)). An additional difference between the Elmegreen & Burkert (2010) work and ours is that they take the dissipation timescale to be the perpendicular crossing time, such that $\gamma_{dis} \equiv t_{perp}\omega = (H/\sigma)(\pi G\Sigma_g/\sigma) = (\sigma^2/\sigma\pi G\Sigma_{disc})(\pi G\Sigma_g/\sigma) = \delta$, while we assume γ_{dis} to be a constant $\approx 1-3$. As a result, their solution at low accretion rates (and low δ) is not as over-stabilized as ours (but still more stable than a case with a higher accretion rate). However, they do not directly discuss the case where the accretion is low enough that the disc is over-stabilized at late

times, where we have shown that a decreasing ξ_i or D may allow for a late marginally-unstable configuration.

Klessen & Hennebelle (2010) conclude that there is more than enough accretion energy to drive the observed turbulence at high redshift. They make a simple comparison between the energy input rate $0.5\dot{M}_{in}V_{in}^2$ and the dissipation rate, and apply relevant numerical values in different situations to see which one is larger. In the case of clumpy discs at high redshift, it seems that they used unrealistic numerical values. Using appropriate values in their eq. (8), either for individual clumps or for the whole disc, yields that a high conversion efficiency is required, $\xi_i \approx 1$, rather than $\xi_i \ll 1$ according to their estimate. This would imply that there is barely enough energy in the in-streaming for being the sole driver of the turbulence, in agreement with our results. In particular, for the case of single clumps, the result in their eq. (16) should be compared with the SFR in a single $10^8\,\mathrm{M_\odot}$ clump, i.e. $\sim 1\,\mathrm{M_\odot\,yr^{-1}}$, not with that of the whole disc, $10 - 50\,\mathrm{M_\odot\,yr^{-1}}$. Such a proper comparison would give a conversion efficiency of ≈ 1. For the case of the whole disc, their choice of $\sigma_{3D} = 30\,\mathrm{km\,s^{-1}}$ is too low in the sense that it corresponds to $V/\sigma \gtrsim 10$. A choice of $V/\sigma \approx 5$ would require an efficiency that is ≈ 10 times higher than their estimate. In addition, there is a factor of ≈ 3.2 missing in the transition from their eq. (15) to eq. (16), since $(30\,\mathrm{km\,s^{-1}})^3/(2G) \approx 3.2\,\mathrm{M_\odot\,yr^{-1}} \neq 1\,\mathrm{M_\odot\,yr^{-1}}$.

Krumholz & Burkert (2010) and Forbes et al. (2011) solve the evolution equations for a thin axisymmetric disc under the assumption of self-regulated marginal instability. The turbulent energy input rate is driven in their model by the inflow rate within the disc (corresponding to a fixed ξ_m in our model), and it is dissipated on a crossing timescale. They solve the evolution equations for the required inflow rate to keep $Q = 1$, which is very similar to the possibility of letting γ_{inf} change in our model (and as assumed in Cacciato et al. 2011), keeping all other factors fixed (Section 4.3). The Krumholz & Burkert (2010) solution is very similar to ours in case II where we force $Q = 1$ (Section 4.1). However, in their fiducial solution for $z \approx 2$ discs they obtain (see their eq. (45)) an inflow rate that has to be roughly six times higher than the star-formation rate, or in our formalism $\gamma_{inf}/\epsilon_{SF} \approx 6$. Similarly, the resulting disc inflow rate at $z \approx 2$ in the Forbes et al. (2011) fiducial disc, where they also assume outflows with $\eta = 1$, is higher than the SFR in disc (Forbes, J., private communication). This is another manifestation of our conclusion that the high turbulence at high redshift cannot be solely driven by the direct effect of the incoming streams, and it requires additional contributions from an intense inflow within the disc, and possibly a depletion by outflows. We conclude that the Krumholz & Burkert (2010) and Forbes et al. (2011) results also agree with the other studies reviewed in this section.

6 SUMMARY AND CONCLUSIONS

We have developed an analytic model that describes certain aspects of the evolution of galactic gas discs in a cosmological context. Specifically, we addressed the possibility that the streaming of external gas into the discs, which provides new fuel for disc instability and star formation, also directly drives turbulent motions by converting the in-

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[6] Note that the factor $\beta^{-1}\gamma^{-2}$ in eq. (10) of Genzel et al. (2008) should be corrected to $\sqrt{\beta^{-1}\gamma^{-2}}$ (Genzel, R., private communication).

streaming kinetic energy into disc turbulence. The results of the model agree with the observations of gas-rich $z \sim 2$ galaxies, where $\sigma/V_{\rm rot} \sim 0.2-0.3$ and $Q \approx 1$. In the absence of self-regulation, with all the parameters fixed, we find that the ratio of turbulent to rotation velocity $\sigma/V_{\rm rot}$ remains constant in time independently of the accretion rate, and that discs evolve from violent instability at high redshift toward stability at low redshift. The constancy of $\sigma/V_{\rm rot}$ with redshift does not agree with the trend suggested by observations. In a second case, where the discs are assumed to be self-regulated to marginal instability about $Q \approx 1$ as a result of a duty cycle in the instability and the associated star formation and outflows, $\sigma/V_{\rm rot}$ is found to decline with cosmic time in better accord with the observations. In this case the duty cycle declines at late times toward long periods of stability separating short episodes of instability. Since there is no evidence at low redshift for a strong increase in the mean timescale for gas consumption into stars, the small duty cycle seems to be observationally unfavourable.

At $z \sim 2$, we conclude that only if the conversion efficiency of in-streaming kinetic energy to turbulence is high, i.e. close to unity, the direct driving of turbulence by the external accretion can be sufficient. Still, unless the SFR efficiency is on the high side of the common estimates, or the turbulence decay rate is much slower than the dynamical time, this scenario relies on disc depletion caused by either galactic winds or inflows within the disc, or both. At low redshift, the driving of turbulence by the inflowing gas may actually be too much if the conversion efficiency remains high and the instability and disc depletion are assumed to be continuously 'on' (see similar conclusions in Dekel et al. 2009b). A conversion efficiency or a duty cycle that decline with time may help recover the evolutionary trends.

We wish to emphasize the role played by the depletion from the disc in allowing a high velocity dispersion in the remaining disc gas. This depletion is a natural result of star formation, galactic winds, and inflows within the disc. With less gas in the disc, the same energy input would drive a higher velocity dispersion. In particular, if stellar feedback and/or AGN feedback drive massive outflows, the disc turbulence will be enhanced even if there is no direct energy injection from the feedback source into the remaining ISM. If some fraction of the outflows is recycled back into the disc, this may add to the direct stirring up of turbulence by accretion.

In a companion paper (Cacciato et al. 2011), we neglect the direct contribution of the in-streaming energy, assuming that the disc turbulence is powered by the inflow within the disc, which is intimately coupled to the self-regulation of the disc instability at $Q = 1$. We find there that the instability is unavoidable at high redshift, because of the intense accretion that maintains a high gas fraction, and that the discs are driven to $Q > 1$ at low redshift, primarily due to the growing dominance of the stellar component. In this model $\sigma/V_{\rm rot}$ tends to decline at late times. This is in qualitative agreement with the trends we find here when we impose $Q \sim 1$.

In Cacciato et al. (2011) we considered both gas and stars in a two-component disc instability analysis, while here we limited the analysis to a one-component gas disc, in order to allow for an analytic solution. Our results there showed that in order to maintain marginal instability, the presence of a 'hot' stellar component implies a lower gas velocity dispersion than in the one-component case. This indicates that the inclusion of stars in the model considered here would have made it even harder to properly suppress the turbulence driven by external accretion and reproduce marginal instability at low redshifts.

To summarize, our model suggests that turbulence driven by cosmological in-streaming may account for the high turbulence observed in $z \sim 2$ discs, but only if the coupling between this inflow and the disc is high. On the other hand, at low redshift our model is in tension with observations unless the conversion efficiency of the in-streaming energy evolves in a certain way to a low value. Thus, cosmological in-streaming could in principle have an important role in driving turbulence in galactic disks, but for this to be the primary driver, and to hold throughout cosmic time, the energy conversion efficiency between inflow and disk has to be fine-tuned.

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APPENDIX A: CONDITIONS FOR A QUASI-STEADY STATE SOLUTION

For the solution of eq. (7) with the assumption $\dot{M}_{\text{sink}} = M_g \tau^{-1}$ to be eqs. (8) and (9), it is required that $\dot{M}_{\text{cosmo}}$ can be treated as a constant, i.e. that it changes slowly with respect to the typical timescale of the solution, namely

$$\left| \frac{d\dot{M}_{\text{cosmo}}/dt}{\dot{M}_{\text{cosmo}}} \tau \right| < 1. \tag{A1}$$

We can write

$$\frac{d\dot{M}_{\text{cosmo}}/dt}{\dot{M}_{\text{cosmo}}} = \dot{M}_{\text{cosmo}}^{-1} \left(\frac{\partial \dot{M}_{\text{cosmo}}}{\partial t} + \frac{\partial \dot{M}_{\text{cosmo}}}{\partial M} \frac{dM}{dt} \right) \tag{A2}$$

since $\dot{M}_{\text{cosmo}}$ changes with cosmic time for a given halo mass, while it also changes for a given halo as it builds up its mass.

For ΛCDM cosmology we can write approximately $\dot{M}_{\text{cosmo}} \propto (1 + z)^{2.2} \propto t^{-1.7}$, where t is cosmic time (Neistein et al. 2006; Genel et al. 2008). Therefore, the first term equals $\dot{M}_{\text{cosmo}}^{-1} \partial \dot{M}_{\text{cosmo}}/\partial t \approx -1.7 t^{-1}$. Further, we can replace dM/dt in the second term with $\dot{M}_{\text{cosmo}}$, and define $t_{\text{acc}} \equiv \partial \dot{M}_{\text{cosmo}}/\partial M$ as in Bouché et al. (2010), to obtain from eq. (A1) the condition

$$\tau^{-1} > \left| -1.7 t^{-1} + t_{\text{acc}}^{-1} \right|. \tag{A3}$$

Since the two terms on the right-hand side have opposite signs, considering each of them separately gives the most stringent conditions that are required for $\dot{M}_{\text{cosmo}}$ to be considered a constant when solving eq. (7). When we consider only the growth of $\dot{M}_{\text{cosmo}}$ that accompanies the halo growth, eq. (A3) becomes $t_{\text{acc}} > \tau$. This condition is discussed in Bouché et al. (2010), where it is found to hold for $z \lesssim 7$. When, instead, we consider only the direct dependence of $\dot{M}_{\text{cosmo}}$ on cosmic time, a stronger condition is obtained, as eq. (A3) becomes $t > 1.7\tau$. This is similar to the condition $t \gg \tau$ that is required to obtain the solution in eqs. (10)-(12).

Therefore, we only have left to consider the relation between t and τ. Since τ is shorter than the star-formation timescale t_{SF} (as it includes also inflows and outflows), we can compare t to t_{SF} as an even more stringent requirement. In our model, $t_{\text{SF}} = t_{\text{dyn}}/\epsilon_{\text{SF}}$, which can be connected to the cosmic time if the disc radius is roughly a constant fraction λ of the virial radius, via the virial dynamical time: $t_{\text{dyn}} \approx 0.1\lambda t$. For $\lambda \approx 0.05$, and our fiducial $\epsilon_{\text{SF}} = 0.02$, we obtain $\tau < t_{\text{SF}} \approx 0.25t$. Even if the star-formation timescale does not become shorter at higher redshift as we assume, and is instead roughly constant $t_{\text{SF}} \approx 2\,\text{Gyr}$ (Genzel et al. 2010; Daddi et al. 2010), τ is still short enough compared to the cosmic time during the epoch of interest here $z \leqslant 2$, or even at somewhat higher redshifts.

In the discussion above, we have referred to the evolution of the average accretion rate over an ensemble of halos. In principle, the accretion rate of any individual galaxy may differ significantly from this average, thus preventing it from reaching the steady-state solution. However, both theoretical arguments and observational evidence indicate that this is likely not the case for the majority of galaxies. From a theoretical point of view, an upper limit on the variations of the accretion rates in individual galaxies is inferred from the scatter around the mean accretion rate found in cosmological simulations, which is $\approx 0.2 - 0.3$dex for either dark matter (Genel et al. 2008) or baryons (Dekel et al. 2009a). Indeed, galaxies in cosmological hydrodynamical simulations are often found to follow the steady-state solution for long cosmological times (e.g. Ceverino et al. 2010; Genel et al. 2012; Dekel & et al. 2012). On the observational side, galaxies populate a rather narrow region in the SFR-stellar mass plane with an overall scatter of ≈ 0.3dex (e.g. Daddi et al. 2007; Noeske et al. 2007; Whitaker et al. 2012; Salmi et al. 2012). Such a tight relation could not be explained in the presence of large SFR variations.

We now consider the importance of mergers in driving galaxies out of the steady-state solution. It is clear that our model does not apply to galaxies after experiencing a major merger event, following which they go out of equilibrium in various ways, and in particular may have their disks transformed into spheroids. However, major mergers are not frequent, as minor mergers and smooth accretion contribute most of the mass in structure formation (e.g. Genel et al. 2010). Minor mergers may trigger the formation of a bar, which is however a 'slow' secular process that does not affect the processes we consider in this paper to a large degree, at least during the violent disk instability phase. They also do not produce significant star-formation bursts (e.g. Jogee et al. 2009). Furthermore, cosmological simulations show that about half of the external galaxies that come in as minor mergers join the disk and share its kinematics without affecting its steady state (Mandelker & et al. 2012). However, even if we consider all mergers with mass ratios $> 1 : 10$ as 'disruptive' as far as our model assumptions go, we should note that galaxies with a stellar mass of $10^{12}\,\text{M}_\odot$ $(10^{11}\,\text{M}_\odot)$ at $z = 0$ have probably experienced only ≈ 7 (≈ 1) such mergers since $z = 2$ (Hopkins et al. 2010). Considering this frequency against the number of galaxy dynamical times elapsed within this cosmic time (> 100), we can expect most galaxies not to be found in an out-of-equilibrium state caused by mergers.

APPENDIX B: CONVERSION OF POTENTIAL ENERGY DURING DISC INFLOWS

The conversion of potential gravitational energy into turbulent energy $(\xi_m > 0)$ is justified by assuming the gas joins the outskirts of the disc and generates turbulence as it migrates to the bulge through the disc (Krumholz & Burkert 2010; Cacciato et al. 2011), rather than by hitting the disc from above. In this picture, the inflow rate γ_{inf} adjusts itself to give $Q = 1$ via a self-regulation loop, since the inflow is a result of the instability in the disc.

In principle, that released energy can be transferred in part to the dark matter component, but we do not address this explicitly here, as the dark matter is assumed to be subdominant inside the disc. The 'immediate' radiation of this energy is not considered, because such losses are already encapsulated in the dissipation timescale of the turbulence.

However, when the energy is transferred to the gas mass that stays in the disc, it can also in principle be in the form of rotational energy. The purpose of the following analysis is to show that under the standard picture of angular momentum loss, where a small mass gains most of the lost angular momentum, the conversion to rotational energy is negligible.

The angular momentum loss rate is $\dot{M}_{\mathrm{inf}}RV$. Let us assume that angular momentum is transferred into a fraction a of the remainder of the disc mass. That fraction of mass that is pushed out by obtaining additional angular momentum has radius R_a and velocity V_a that can be different from that of the disc as a whole. This can be represented by writing

$$\dot{M}_{\mathrm{inf}}RV = aM_{\mathrm{g}}\frac{\partial(R_aV_a)}{\partial t} = aM_{\mathrm{g}}\dot{R}_aV_a + aM_{\mathrm{g}}R_a\dot{V}_a \quad (B1)$$

$$= aM_{\mathrm{g}}\dot{R}_aV_a + aM_{\mathrm{g}}R_aV_a\frac{\dot{R}_a}{2R_a} = \frac{3}{2}aM_{\mathrm{g}}\dot{R}_aV_a,$$

where the penultimate equality is based on the equality $\dot{V}_a = 0.5\sqrt{GM_{\mathrm{g}}}\dot{R}_a/(R_a)^{1.5} = 0.5V_a\dot{R}_a/R_a$, which stems from the assumption $V_a = \sqrt{GM_{\mathrm{g}}/R_a}$. When the mass aM_{g} gains the angular momentum, it is accompanied by an energy gain rate that can be written as follows:

$$\frac{\partial(GaM_{\mathrm{g}}^2/R_a)}{\partial t} = \frac{GaM_{\mathrm{g}}^2}{R_a^2}\dot{R}_a \quad (B2)$$

$$= aM_{\mathrm{g}}\frac{\dot{R}_a}{R}\frac{GM_{\mathrm{g}}}{R}\left(\frac{R}{R_a}\right)^2$$

$$= \frac{2}{3}\frac{V}{V_a}\dot{M}_{\mathrm{inf}}V^2\left(\frac{R}{R_a}\right)^2.$$

This quantity should be compared with the energy that is available from the inflows, i.e. $\dot{M}_{\mathrm{inf}}V^2$. The crucial factor is a, which comes in to the final expression in eq. (B2) via $(R/R_a)^2$. If the angular momentum is given to a small amount of mass ($a \ll 1$) that is expelled to large distances due to the angular momentum it acquires, namely $R \ll R_a$, then the contribution of the inflows to rotational energy in the remainder of the disc is small, and most of the energy goes to turbulent energy ($\xi_m \approx 1$). However, if the angular momentum of the inflowing gas is distributed evenly across the whole disc, then the rotational energy gain that is associated with this angular momentum gain is comparable to the energy that is released by the inflow, namely little energy is left for driving turbulence in the disc and then $\xi_m \ll 1$.

APPENDIX C: APPROXIMATIONS FOR PARAMETER NUMERICAL VALUES

The following assumptions are used throughout in order to evaluate numerical values for different quantities.

• The star-formation efficiency per dynamical time is small: $\epsilon_{\mathrm{SF}} \ll 1$.

• The wind mass-loading factor is not very large: $\epsilon_{\mathrm{SF}}^{-1} \gg (1 + \eta)$.

• Turbulent energy dissipation is much faster than star formation: $t_{\mathrm{dis}} \approx t_{\mathrm{dyn}}$.

• Angular momentum loss is *not* much faster than star formation: $\gamma_{\mathrm{inf}} \sim \epsilon_{\mathrm{SF}}$. Also, one can derive $\gamma_{\mathrm{inf}} \approx 0.01$ by comparing eqs. (19)-(21) in Dekel et al. (2009b) to the definition of $\dot{M}_{\mathrm{inf}}$ in Section 2.2.

• When gas is accreted at the virial radius, it has potential energy of $\approx 3V_{\mathrm{vir}}^2$ relative to the halo centre (for NFW haloes), and kinetic energy per unit mass of $\approx 0.5V_{\mathrm{in}}^2$. We assume that most of that potential energy is lost to radiation (supported by simulations; Dekel et al. 2009a). Thus, at the arrival to the disc, the potential energy between the disc edge and the centre is $\approx V_{\mathrm{vir}}^2$ and the kinetic energy is still $\approx 0.5V_{\mathrm{in}}^2$. Dekel et al. (2009a) estimate $V_{\mathrm{in}} \approx (1.5 - 2)V_{\mathrm{vir}}$. We assume in addition $V_{\mathrm{rot}} \approx (1 - 1.5)V_{\mathrm{vir}}$. Thus, $u \equiv V_{\mathrm{rot}}/V_{\mathrm{in}} \approx 0.75 \pm 0.25$. We note that we thus assume roughly equal amounts of energy of the incoming gas, at the arrival to the disc, in kinetic and potential form: potential energy of $\approx V_{\mathrm{vir}}^2$ and kinetic energy of $\approx 0.5V_{\mathrm{in}}^2 = 0.5(1.5 - 2)^2V_{\mathrm{vir}}^2 \approx (1 - 2)V_{\mathrm{vir}}^2$.